

# The Special Theory Of Relativity

## Relativity Of Time And Space

### Abstract

This paper will provide an introduction to the special theory of relativity as well as the most important knowledge about inertial reference frames, the simultaneity and the properties of light.

### Introduction

The approach is as less as possible mathematical and tries to convey the effects of the relativity more by examples and experiments. The paper does not expect much physical knowledge from the reader. It will provide the most important knowledge before it comes to the core of the paper the relativity of time and space. To obtain a proper understanding of the special theory of relativity it is reasonable to have a look at Newton's First Law and his inertial frames of reference, we will clarify and define the simultaneity of events and we have to be acquainted with some properties of light. With this as a base and equipped with a vivid mind there should be no serious difficulties left on our way.

### The Inertial Frames Of Reference

Approaching this theme properly it is necessary to introduce the term of the inertial frame of reference. Achieving this we are going to take a look at the first Law of Newton which predicts that :

If no force acts on a body then the body's velocity cannot be changed that is the body cannot be accelerated.

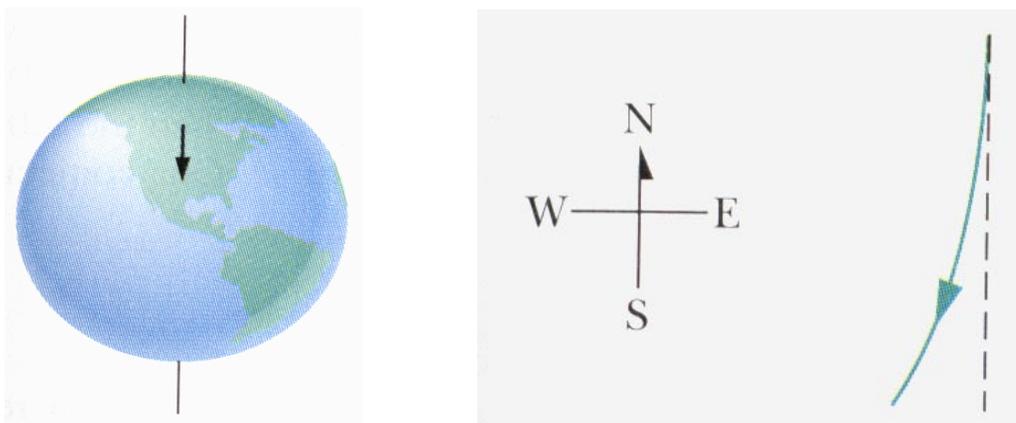


Fig. 1

This means e.g. if one playing puck sends his puck (see Fig. 1) along a short line of frictionless ice then the puck will not change its speed or direction. But suppose we made a strip very long from the north to the south pole. Then our player on the ground would find out that the puck accelerates toward the west as it moves south. Yet our player would not be able to find any force that causes the westward acceleration of the sliding puck. The surprising westwards acceleration of the sliding puck

relative to the line is explained by the rotation of the earth. It follows that the first Law of Newton is not satisfied. To avoid such troubles we define the inertial frame of references or inertial reference frame in that way saying that Newton's First Law only in this inertial reference frames is true. That means: in that moment we leave one frame by starting to move relative to it the law can no longer be applied due to the fact that we change the inertial reference frame. In our example we have the puck in one frame and the surface of the earth in another. Anyway for an observer in the outerspace the puck obeys the first Law of Newton because he is still in the same inertial reference frame.

### Closer Look at Simultaneity

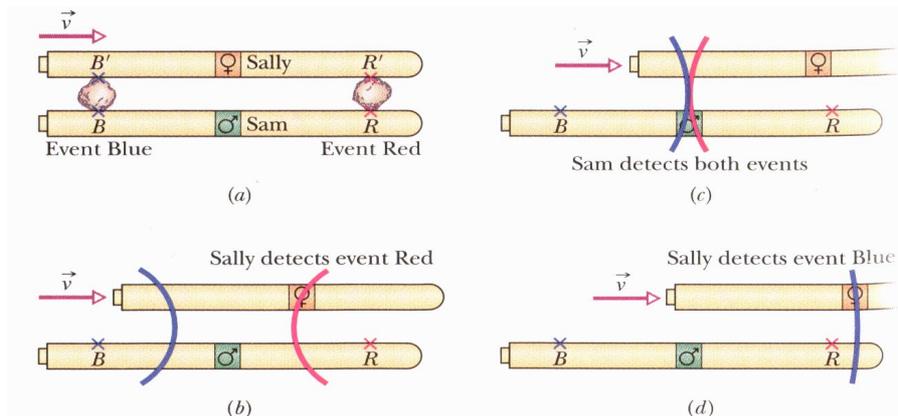


Fig. 2

Imagine that one observer (Sa) notes two independent events occurring at the same time (as depicted in Fig. 2). Moreover suppose that another observer called Sally is moving with a constant speed relatively to Sam. Under these conditions we are going to find out that the same two events which happened at the same time for Sam at least will be recorded at different times by Sally.

Let us suppose that these two events create two wavefronts as well as two marks on each spaceship in the moment when these two events happen.

Then after that episode Sam will say that both waves (the red and the blue one) reached him at the same time and by measuring the distances of the marks he will find out that he was exactly halfway between them (when the lightwaves reached him).

But Sally will state that the light from the event red had reached her before the lightwave from the event blue did because of the same direction in which she and the event-blue-wave moved. From the marks placed on the ship she can conclude that her position was also halfway between them.

These reports do not agree but both are just correct.

From now on we define two events to be simultaneous if an observer stationed midway between them detects them at the same moment.

So if two observers are in relative motion they will not agree as to whether two events are simultaneous. If one observer finds them to be simultaneous the other generally will not. These considerations are going to be important if we want to start two clocks simultaneously. Doing so will become necessary as we will see.

Finally to synchronize two clocks that are some distance apart requires a special procedure. Making sure the clocks are synchronized assuming they are both accurate is to start them together. How can we do that? We could for example attach a photocell to each clock so when a flash of light reaches the clock it begins running.

## The Light and Its Properties

Before we turn to the relativity it is useful to take a closer look at the properties of light. Since the middle of the 19<sup>th</sup> century it was established that light has some properties of waves. Maxwell a physicist discovered that his equations predicted: there could be waves made up of electric and magnetic fields and the speed of these waves deduced from experiments on how these fields link together would be  $c=186,300$  miles per second i.e. the speed of light. Accepting this it so far not clear what is waving. For sound e.g. it is common knowledge that medium for the wave of sound is vital. Taking this as a hint it was natural to suppose that light must be just waves in some mysterious material which was called the ether surrounding and permeating everything. Detecting this ether wind the physicist Michelson set up an experiment the so called Michelson–Morley Experiment. The very clever idea for detecting the ether wind which was supposed to blow due to the movement of the earth around the sun can be well explained with the following example: Suppose we have a river of width  $w$  and two swimmers who both swim at the same speed  $v$ . The river is flowing at a steady rate. The swimmers start both at the same point on one bank. One (swimmer A) swims directly across the river to the closest point on the opposite bank then turns around and swims back. The other (swimmer B) stays on one side of the river swimming upstream a distance  $w$  (the width of the river) then swims back to the start. Who wins? It is of course the swimmer B who wins the race. The swimmer A has the problem that while he is swimming across the river the stream of the river tries to carry him downstream. Thus the resulting length he has to cover is more than just twice the width  $w$ . The swimmer B instead goes downstream with the flow and upstream against them so the flow can be neglected and he just swims twice the length of the river. That is to say the choosing of the inertial reference frame is important if we are going to make statements about the covered length of the swimmers. Considering the covered length from the land or the bank we will discover that the length are both equal. But choosing the steadily flowing river as a reference frame it becomes clear which length is really swam by the swimmers (as we have seen).

Anyway, the idea was to detect the speed of light relative to the ether.

This must mean if you could measure the speed of light accurately enough you could measure the speed of light travelling upwind and compare it with the speed of light travelling downwind and the difference of the two measurements should be twice the windspeed.

Michelson invented an interferometer to measure the speed of light pulses directed at an angle of 90 degrees. Now if there is an ether wind blowing someone looking through the telescope should see the halves of the two half–pulses to arrive at slightly different times since one would have gone more upstream and back one more across stream in general. Just like in the river example.

The result of the experiment was that the lightbeams did not have any changes in wavelengths. So the conclusion drawn by Einstein was that the concept of the ether was wrong.

It became proven in the nineteen–sixties that this speed of light is always the same. It is independent from the inertial frame of reference we always measure the same speed of light. E.g. the speed of light emitted from a flying aircraft is measured from the earth not the speed of light plus the speed of the aircraft but just the speed of light.

Summing up the essential property of the light we need for the further lecture is: light always spread with the same constant speed.

## Relativity of Time

We have accepted that light always travels at speed  $c$  relative to the observer.

However it was Einstein who concluded that no kind of observation at all even measuring the speed of light to any accuracy you like would help find out if you was "really at rest". This implies of course that the concept of being "at rest" is meaningless. The laws of physics are the same in any inertial frame and in particular any measurement of the speed of light in any inertial frame will always give  $c=186,300$  miles per second.

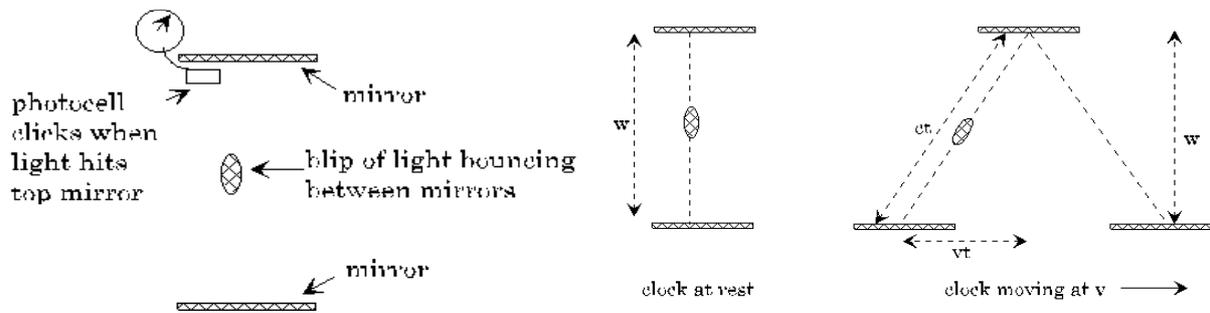


Fig. 3

Clarifying the relativity of time we do a so called Gedankenexperiment. We imagine a clock like illustrated in the Fig. 3. A pulse of light leaves the lightsource B travels vertically upward is reflected by the mirror and then is detected at the source. So by counting these arrivals we get a well working clock based on the speed of light. Further we can imagine that we take two of these clocks and synchronise them by a signal sent out in the middle of these clocks. Now we leave one clock (clock 1) in rest and move the other one (clock 2) with a constant speed. As we can see travels the lightbeam because of the motion of clock 2 a longer way than the lightbeam in clock 1. The result is that the clock 2 runs obviously slower than the clock 1. Hence it follows that the time in inertial frames of rest which are in motion relative to each other goes slower. This effect is called time dilation . To became more acquainted with this phenomenon suppose now the clock 1 is with a person A on a wagon. This wagon moves with a constant speed  $v$ . The pulse of light takes time  $t$  to get from the bottom mirror to the top mirror as measured by person A and person B who is standing by the track. From B's point of view the length of the "zig" from the bottom mirror to the top mirror is necessarily  $ct$  since that is the distance covered by any blip of light in time  $t$ . Meanwhile the wagon has moved down the track a distance  $vt$ . We have a right-angled triangle with hypotenuse  $ct$  and shorter sides  $vt$  and  $w$ .

By Pythagoras then

$$(ct)^2 = (vt)^2 + w^2$$

hence

$$t^2(c^2 - v^2) = w^2$$

dividing by  $c^2$  yields

$$t^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{w^2}{c^2}$$

and by taking the squareroot of each side we conclude that B measures the time between the clicks of A's clock as:

$$t_D = t \sqrt{1 - \frac{v^2}{c^2}}$$

This means that B sees A's light clock to be going slow—a longer time between clicks—compared to his own identical clock. Obviously the effect is not dramatic at real railroad speeds. Nevertheless the effect so called time dilation is real and can be measured. The dilation is even so large that it has to be taken into account in today's satellite navigation.

However it is important to realize that the only reason we prefer a light clock as opposed to some other kind of clock is that its motion is very easy to analyse from a different frame. A could have a collection of clocks on the wagon and would synchronize them all. Nevertheless B will see the synchronized light clock and wristwatch next to each other and of course note that the wristwatch is also running slow by the factor  $\sqrt{1-v^2/c^2}$ . In fact all the clocks are slowed down by this factor according to B.

But this is not the whole story –we must now turn everything around and look at it from A’s point of view. His inertial frame of reference is just as good as B’s (as we have learned before). He sees B’s light clock to be moving at speed  $v$  (backwards) so from A’s point of view the light blip takes the longer zigzag path which means B’s clock runs slower than his. That is to say each of them will see the other to have slower clock. This time dilation has been verified in the early sixties by flying very accurate clocks around the world on jetliners and finding they register less time by the predicted amount than identical clocks left on the ground.

### **The length contraction (Fitzgerald Contraction )**

As it we have seen is a relation existing between the speed and the time. But what about space? The answer of this question is almost given. Anyway let us suppose A’s clock is equipped with a device marking the track once a second. What distance will be between the marks? From A’s point of view this is pretty easy to answer. He sees the track passing under the wagon at  $v$  feet per second say to choose convenient units of one second so the notches will of course be  $v$  feet apart. But B sees things differently. He observes A’s clocks to be running slow, so he will notice the marks on the track at intervals of  $1/\sqrt{1-v^2/c^2}$  seconds. Since A agrees with B that the relative speed of the wagon and the track is  $v$  he will assert the notches are not  $v$  feet apart, but  $v/\sqrt{1-v^2/c^2}$  feet apart a greater distance. Who is right? It turns out that B is right because the notches are in his frame of reference so he can walk over to them with a tape measure or whatever and check the distance. This implies that as a result of his motion A observes the notches to be closer together by a factor  $\sqrt{1-v^2/c^2}$  than they would be at rest. To get a mathematical approach we just have to take our time dilation formula and try to substitute the time  $t$  with  $l=vt$  or  $t=l/v$  and we can write

$$\frac{l_c}{l} = \frac{vt_D}{vt} = \sqrt{1 - \frac{v^2}{c^2}}$$

or

$$l_c = l \sqrt{1 - \frac{v^2}{c^2}}$$

This is called the Fitzgerald contraction and applies not only to the notches but also to the track and to B –everything looks somewhat squashed in the direction of motion!

Thus we realize that terms like time and space which we consider as constant due to our experiences and perceptions are relative. It is almost impossible to obtain a comprehensive or real understanding of these effect. Because we are grown up without perceiving these phenomenon of time dilation. It appears strange that the flow of time changes if we start to move. In opposite to gravity of the earth for e.g. which we experience since our earliest childhood and which is even experienced since the dawn of mankind. Nevertheless we do not now exactly what gravity is but it appears much more common and usual to us then time dilation and length contraction. Whatever even if it is probably difficult to imagine I hope the paper contributed to a understanding of this part of our nature.

## References

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